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A CORRELATION FOR HEAT TRANSFER BY NATURAL CONVECTION FROM HORIZONTAL CYLINDERS THAT ACCOUNTS FOR VISCOUS DISSIPATION

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Abstract Eight previously published correlation equations plus one new correlation for heat transfer by natural convection from horizontal isothermal cylinders are tested against a fairly extensive body of experimental data culled from the literature for $10^{-8} < Ra < 10^8$ and $0.7 < Pr < 4 \times 10^4$. The new equation, which represents the Nusselt number as a function of the Prandtl and Rayleigh numbers plus an additional dimensionless parameter that accounts for viscous dissipation, is shown to correlate the experimental data more accurately than does any one of the eight previously published equations. It is concluded that viscous dissipation may not be neglected in all cases of natural convection from horizontal cylinders, and further, that the inclusion of a viscous dissipation term in certain related problems, such as natural convection in porous media, may lead to more accurate correlation equations.

NOMENCLATURE

- c, specific heat at constant pressure;
- d, diameter;
- E, error summary statistic (see postscripts);
- e, percentage error, percentage error of Nu is defined as $e = [Nu(\text{calc}) - Nu(\text{obs})]100/Nu(\text{obs})$;
- F, f, functions;
- Ge, Gebhart number, $g\beta L/c$;
- Gr, Grashof number, $g\beta L^3\Delta t/\nu^2$;
- Ec, Eckert number, $V^2/c\Delta t$;
- g, gravitational acceleration (9.80665 m s^{-2}); a function;
- h, heat transfer coefficient;
- k, thermal conductivity;
- L, characteristic length;
- N, number of data points in a data set or subset;
- Nu, Nusselt number, hL/k;
- Pr, Prandtl number, $\mu c/k$;
- Ra, Rayleigh number, $\rho g\beta L^3 c\Delta t/\nu k$;
- Re, Reynolds number, VL/ν;
- t_b, bulk fluid temperature;
- t_s, surface temperature;
- V, velocity;
- x, distance.

Greek symbols

- β, coefficient of thermal expansion;
- Δt, temperature difference, $t_s - t_b$;
- μ, dynamic viscosity;
- ν, kinematic viscosity.

Postscripts

- ave, average value, e.g. $E(\text{ave}) = \Sigma e_i/N$;
- calc, value calculated using a correlation equation, e.g. $Nu(\text{calc})$;
- max, maximum value, e.g. $E(\text{max})$ = largest of $e_1, e_2, \ldots, e_N$;
- min, minimum value, e.g. $E(\text{min})$ = smallest of $e_1, e_2, \ldots, e_N$;
- obs, value based on experimental data, e.g. $Nu(\text{obs})$;
- rms, root mean square value; e.g. $E(\text{rms}) = (\Sigma[e_i]^2)^{0.5}$.

Miscellaneous

- #, indicates a number used to identify a subset of data from a source in Table 2.

INTRODUCTION

THE FIRST eight equations in Table 1 constitute a list of previously published empirical and semi-empirical correlation equations for heat transfer by natural convection from an infinitely long horizontal isothermal cylinder immersed in a body of fluid that is infinite in extent. A cursory examination of these eight equations reveals that they do not agree closely with one another; for example, for cylinders in air and $Ra = 1$, the value of Nu calculated from equation (8) exceeds that calculated from equation (6) by approximately 50% of the value calculated by equation (8). Furthermore, as might be expected, these equations do not represent experimental data obtained from various sources equally well, particularly at low Rayleigh numbers ($Ra < 1$).

Table 1. Correlation equations for heat transfer by natural convection from horizontal isothermal cylinders

Equation (1)	Rice [1]	
	$Nu = 2/\ln[1+2/(0.47\,Ra^{0.25})]$.	
Equation (2)	Senftleben [2]	
	$Nu = 2\{1-([1+XY]^{0.5}-1)/(XY)\}/X$,	
	$X = \ln[1+4.5/Ra^{0.25}]$, $\quad Y = Ra^{0.25}/0.033$.	
Equation (3)	Kyte [3]	
	$Nu = 2/\ln[1+7.09/Ra^{0.37}]$,	$10^{-7} < Ra < 10^{1.5}$,
	$Nu = 2/\ln[1+5.01/Ra^{0.26}]$,	$10^{1.5} < Ra < 10^{9}$.
Equation (4)	van der Hegge Zijnen [4]	
	$Nu = 0.35+0.25\,Ra^{0.125}+0.45\,Ra^{0.25}$,	$10^{-7} < Ra < 10^{9}$.
Equation (5)	Tsubouchi and Masuda [5]	
	$Nu = 0.36+0.048\,Ra^{0.125}+0.52\,Ra^{0.25}$,	$10^{-6} < Ra < 10^{9}$.
Equation (6)	Churchill and Chu [6]	
	$Nu = 0.36+0.518\,Ra^{0.25}[1+(0.559/Pr)^{9/16}]^{-4/9}$,	$10^{-6} < Ra < 10^{9}$.
Equation (7)	Morgan [7]	
	$Nu = 0.675\,Ra^{0.058}$,	$10^{-10} < Ra < 10^{-2}$,
	$= 1.020\,Ra^{0.148}$,	$10^{-2} < Ra < 10^{2}$,
	$= 0.850\,Ra^{0.188}$,	$10^{2} < Ra < 10^{4}$,
	$= 0.480\,Ra^{0.250}$,	$10^{4} < Ra < 10^{7}$,
	$= 0.125\,Ra^{0.333}$,	$10^{7} < Ra < 10^{12}$.
Equation (8)	Raithby and Hollands [8]	
	$Nu^{3.337} = [2/\ln(1+2.59/C(l)\,Ra^{0.25})]^{3.337}+[0.72\,C(t)\,Ra^{1/3}]^{3.337}$,	$10^{-2} < Ra < 10^{12}$
	$C(l) = (2/3)/[1+(0.49/Pr)^{9/16}]^{4/9}$,	
	$C(t) = \min(0.15, 0.14\,Pr^{0.084})$.	
Equation (9)	$Nu = 0.400\,Pr^{0.0432}\,Ra^{0.25}+0.503\,Pr^{0.0334}\,Ra^{0.0816}$	$10^{-8} < Ra < 10^{8}$
	$+0.958\,Ge^{0.122}/Pr^{0.0600}\,Ra^{0.0511}$.	

All eight previously published equations listed in Table 1 express Nu as a function of Pr and/or Ra. That Nu is a function of only Pr and Ra can be demonstrated by examining the governing set of differential equations for natural convection *under the assumption that viscous dissipation is negligible*. However, Gebhart [9] and Bertela [10] point out that viscous dissipation is not always negligible in natural convection, and it is the objective of this paper to show that a significantly better empirical correlation for heat transfer by natural convection from horizontal isothermal cylinders can be obtained if the Nusselt number is assumed to be a function of a dimensionless parameter that represents viscous dissipation, in addition to Pr and Ra.

Equation (9) in Table 1 is a new empirical correlation for heat transfer by natural convection from horizontal isothermal cylinders in which Nu is a function of Pr, Ra and Ge, where Ge is a dimensionless parameter that represents the influence of viscous dissipation. This new equation and the eight previously published equations listed in Table 1 are evaluated below relative to a fairly extensive body of experimental data described in the next section, following which the rationale behind equation (9) is presented. This order of presentation has been adopted in order to economize on space.

DISCUSSION OF QUALITY AND SOURCES OF DATA

Quality of data

An empirical equation is no better than the experimental data upon which it is based. In the present study, the relative quality of data was judged on the basis of two general criteria: The first criterion relates to the care and precision with which the experimental measurements (heat flux and temperature) were made. The second criterion relates to the degree to which the experiments modelled the phenomenon under study, namely, that of heat transfer by natural convection from an *infinitely* long isothermal cylinder immersed in an *infinite* fluid medium. The first of these criteria is fairly easy to satisfy but the second is not because it requires the establishment experimentally of flow and temperature fields about a cylinder having finite length and immersed in a fluid having finite volume that closely resemble the analogous flow and temperature fields that would occur about a hypothetical infinite cylinder immersed in an infinite medium. The second criterion requires that the volume of fluid surrounding the test cylinder be relatively large and that no spurious disturbances shall affect the natural convection flow; it also requires the elimination of so-called "end effects" at the points of support of the heated test cylinder.

Two techniques have been used by experimenters to satisfy the second criterion: One technique uses cylinders having high length-to-diameter ratios thereby, presumably, rendering the end effects negligible in comparison with the overall measurement for the entire cylinder sometimes calculated corrections for end effects are applied in conjunction with this technique; this method will be referred to hereinafter as the "high aspect ratio technique". The second technique is to use test cylinders having moderate to high aspect ratios but to restrict measurements of heat flux and temperature to a central portion of the cylinder that is relatively far from the ends—sometimes baffles placed near the cylinder supports are used in conjunction with this technique to help create a 2-dim. field; this method will be referred to as the "guarded test section technique".

The authors are of the opinion that the guarded test section technique more closely models the phenomenon under consideration than does the high aspect ratio technique, and they have therefore used data gathered by the guarded test section technique as the primary basis for comparison and correlation purposes. Certain data obtained by the high aspect ratio technique have also been used for these same purposes, but in a secondary role, as is described below.

Sources of data

During the past six years there has been conducted at the University of Hawaii a series of experiments on natural convection from horizontal isothermal cylinders. All these experiments were performed using the guarded test section technique, including baffles. Reference [11], which contains the first published results of this extended study, describes the basic apparatus used in all subsequent experiments. The data so obtained, hereinafter referred to as the UH data, were incorporated into several theses and reports [12–16], and are all listed in [16] together with formulas for calculating fluid properties as functions of temperature. (All fluid properties in the present paper were calculated at the mean film temperature.)

The UH data, together with the data of Kennelly *et al.* [17] (which were also obtained by the guarded test section technique) were combined into a single data set. When exceptional data points* were removed from this combined set, the remainder, called Data Set 1 (abbreviated DS1), contained a total of 250 observations. The data in DS1 were selected as the primary data set for comparison and correlation purposes because the authors believe that these data represent the best available approximation to the hypothetical problem under study (infinite cylinder and medium).

In order to extend the range of application of the results of this investigation to a wider domain than is covered by DS1, the data of Collis and Williams [18] for their two longest cylinders (14 data points) and the data of Tsubuchi and Masuda [19] (84 data points) were added to DS1—the resulting data set which consists of 348 observations, is designated Data Set 2 (DS2). Although the laboratory conditions under which these added data were taken correspond to the high aspect ratio technique (which is not considered optimal) they were included in the present analysis because they exhibit a high degree of internal consistency and fall into a domain of the relevant dimensionless parameters for which no other sources were found.

It is difficult, if not impossible, to establish completely objective criteria whereby to judge the relative quality of experimental data in a previously unexplored domain. Therefore, in order to minimize the possible influence of personal bias on the outcome of the present study, the authors have compiled a set of data that includes all the published data identified in Table 2. This comprehensive data set, which contains 580 data points, is called the Total Data Set (TDS)†. All conclusions drawn in the course of this study have been tested against DS1, DS2 and TDS and the results thereof are presented below.

COMPARISON OF CORRELATION EQUATIONS WITH DATA

Table 3 contains statistics whereby the performance of the nine correlation equations in Table 1 relative to DS1, DS2 and TDS can be compared. On the basis of the entries in Table 3, it is clear that equations (8) and (9) represent DS1 and DS2 more closely, on the whole, than does any of the others. The performance of equation (9) is somewhat superior to that of equation (8) for $Ra > 1$ and markedly so for $Ra < 1$; however, it should be kept in mind that the range of applicability of equation (8) is restricted to $10^{-2} < Ra < 10^{12}$ (see Table 1), and hence the entries in Table 3 for this equation for the lowest Rayleigh numbers must be regarded as being merely informational, and not critical. This same comment applies, to a lesser extent, to equations (5) and (6).

The entries in Table 3 reveal that when the nine correlation equations in Table 1 are tested against TDS, the errors are considerably higher in most cases than the corresponding errors relative to DS1 and DS2. This is not surprising, inasmuch as TDS contains data from diverse sources which do not, it is believed, model a hypothetical infinite cylinder in an infinite medium as faithfully as do the data in DS1 and DS2. Nevertheless, even relative to TDS, equation (9) exhibits the least overall error. The preceding comparisons have led the authors to conclude that the new correlation, equation (9), represents DS1, DS2 and TDS better than does any of the eight previously published equations in Table 1, and is therefore preferable.

The reasoning and statistical analysis that led to the determination of equation (9) is presented in the next section.

* Points within a data set which, on a plot of Nu versus Ra, showed marked deviations from the overall trend for the data set were considered exceptional.

† The raw data for every point in TDS plus formulas for calculating the properties of all fluids encountered are contained in ref. [16]. This reference also contains the computed values of all relevant dimensionless parameters for each point in TDS plus certain statistics not included in this paper because of space limitations.

Table 2. Sources of data and summary of data set identifiers for TDS

Source Range of Pr Range of Ra Range of Ge	#	Test cylinder diameter, d	Fluid	N
Fand [11]	1	1.157 cm	Water	10
$7 \times 10^{-1} < Pr < 3 \times 10^{3}$			20 cSt Oil*	6
$3 \times 10^{2} < Ra < 8 \times 10^{6}$			100 cSt Oil	6
$7 \times 10^{-9} < Ge < 6 \times 10^{-7}$			350 cSt Oil	10
Sharma [12]	1	0.8230 cm	20 cSt Oil	8
$9 \times 10^{1} < Pr < 3 \times 10^{3}$			100 cSt Oil	10
$1 \times 10^{-1} < Ra < 5 \times 10^{2}$			350 cSt Oil	10
$Ge \doteq 5.3 \times 10^{-9}$				
Kerns [13]	1	1.4732 mm	20 cSt Oil	7
$1 \times 10^{2} < Pr < 3 \times 10^{3}$			100 cSt Oil	7
$7 \times 10^{-1} < Ra < 4 \times 10^{3}$			350 cSt Oil	8
$9 \times 10^{-9} < Ge < 2 \times 10^{-8}$	2	2.7750 mm	20 cSt Oil	8
			350 cSt Oil	8
Hessami [14]	1	0.2035 mm	Air	10
$7 \times 10^{-1} < Pr < 3 \times 10^{3}$			20 cSt Oil	15
$1 \times 10^{-2} < Ra < 2$			100 cSt Oil	12
$1 \times 10^{-9} < Ge < 6 \times 10^{-9}$			350 cSt Oil	23
Fox [15]	1	0.0785 mm	Air	12
$7 \times 10^{-1} < Pr < 3 \times 10^{3}$			20 cSt Oil	18
$2 \times 10^{-4} < Ra < 9 \times 10^{-1}$			100 cSt Oil	18
$5 \times 10^{-10} < Ge < 2 \times 10^{-9}$			350 cSt Oil	18
Brucker [16]	1	0.0519 mm	Air	47
$Pr = 7 \times 10^{-1}$				
$6 \times 10^{-5} < Ra < 7 \times 10^{-4}$				
$Ge \doteq 1.5 \times 10^{-9}$				
Kennelly [17]	1	0.2616 mm	Air	25
$Pr = 7 \times 10^{-1}$	2	0.1143 mm	Air	3
$6 \times 10^{-3} < Ra < 2$	3	0.2616 mm	Air	5
$2 \times 10^{-9} < Ge < 2 \times 10^{-8}$	4	0.6907 mm	Air	3
Collins [18]		0.00295 mm		
$Pr = 7 \times 10^{-1}$	1	($l = 18.00$ cm)	Air	7
$3 \times 10^{-8} < Ra < 1 \times 10^{-7}$	2	($l = 4.417$ cm)	Air	7
$6 \times 10^{-11} < Ge < 10 \times 10^{-11}$				
Tsubouchi [5]	1	0.0154 mm	10 000 cSt Oil	7
$5 < Pr < 1 \times 10^{5}$		0.0268 mm	Toluene	9
$1 \times 10^{-6} < Ra < 10$			Spindle oil	10
$5 \times 10^{-11} < Ge < 5 \times 10^{-10}$	2	0.0317 mm	100 cSt Oil	2
		0.0454 mm	10 000 cSt Oil	9
			Toluene	6
			Spindle Oil	
	3	0.0557 mm	100 cSt Oil	2
		0.0627 mm	10 000 cSt Oil	6
			Toluene	14
			Spindle oil	9
Langmuir [19]	1	0.0404 mm	Air	4
$Pr = 7 \times 10^{-1}$	2	0.0691 mm	Air	17
$4 \times 10^{-5} < Ra < 7 \times 10^{-1}$	3	0.1262 mm	Air	4
$4 \times 10^{-10} < Ge < 1 \times 10^{-8}$	4	0.2508 mm	Air	5
	5	0.5100 mm	Air	4
Rice [1]				
$Pr = 7 \times 10^{-1}$	1	4.28 cm	Air	24
$4 \times 10^{3} < Ra < 6 \times 10^{5}$	2	5.56 cm	Air	9
$Ge \doteq 1.3 \times 10^{6}$				
Petavel [20]	1	1.120 mm	Air	24
$Pr = 7 \times 10^{-1}$	2	0.600 mm	Air	9
$3 \times 10^{-1} < Ra < 8$				
$7 \times 10^{-9} < Ge < 3 \times 10^{-8}$				

Table 2. (*contd*)

Source Range of Pr Range of Ra Range of Ge	#	Test cylinder diameter, d	Fluid	N
Griffiths [21]	1	11.40 cm	Air	14
$Pr = 7 \times 10^{-1}$	2	11.51 cm	Air	12
$3 \times 10^{6} < Ra < 6 \times 10^{7}$	3	22.86 cm	Air	9
$2 \times 10^{-6} < Ge < 6 \times 10^{-6}$	4	11.43 cm	Air	6
Wamsler [22]	1	5.900 cm	Air	6
$Pr = 7 \times 10^{-1}$				
$9 \times 10^{5} < Ra < 1 \times 10^{6}$				
$Ge \doteq 1.6 \times 10^{-6}$				
Lemlich [23]	1	1.245 mm	Water	14
$4 < Pr < 5$				
$3 \times 10^{2} < Ra < 2 \times 10^{3}$				
$9 \times 10^{-10} < Ge < 1 \times 10^{-9}$				

* 1 cSt $\equiv 10^{-2}$ m^2 s^{-1}.

Table 3. Summary of the performance of nine correlation equations relative to DS1, DS2 and TDS

	E(rms) by range of Ra*							Within†	
	(1)	(2)	(3)	(4)	all	E(ave)	E(max)	10%	5%
Statistics for DS1									
N	24	111	67	48	250				
Equation (1)	25.3	17.0	9.6	13.6	15.8	2.8	37.4	112	67
(2)	2.7	16.2	20.8	19.7	17.6	−13.3	30.5	66	41
(3)	11.7	22.8	18.7	15.1	19.5	−16.4	31.9	52	36
(4)	9.9	15.5	15.8	13.7	14.8	−7.7	24.2	56	20
(5)	4.2	23.0	22.2	14.5	20.2	−15.0	33.4	63	40
(6)	10.6	28.0	26.4	13.9	24.1	−22.3	37.9	22	12
(7)	8.2	21.5	20.5	17.8	19.6	−16.5	32.7	71	44
(8)	20.1	13.1	6.3	7.7	11.7	4.1	29.4	169	98
(9)	4.8	9.9	8.3	3.5	8.1	−2.7	21.8	203	106
Statistics for DS2									
N	62	159	79	48	348				
Equation (1)	27.5	21.8	10.9	13.6	20.1	9.5	50.4	113	68
(2)	9.2	14.5	19.2	19.7	15.8	−9.5	30.5	134	77
(3)	12.4	19.4	17.3	15.1	17.3	−13.7	31.9	122	72
(4)	24.3	14.5	14.6	13.7	16.6	−1.3	52.6	101	41
(5)	19.7	19.7	20.7	14.5	19.3	−9.8	47.7	130	77
(6)	19.2	24.4	25.4	13.9	22.6	−16.7	44.8	68	32
(7)	8.3	18.3	18.9	17.8	17.0	−12.8	32.7	159	97
(8)	25.6	20.9	8.4	7.7	18.4	10.8	53.8	170	98
(9)	7.0	10.1	7.7	3.5	8.4	−0.7	22.3	270	148
Statistics for TDS									
N	110	218	115	137	580				
Equation (1)	25.9	22.3	12.3	18.5	20.6	6.2	58.4	209	113
(2)	11.7	14.9	19.8	25.9	18.6	−11.8	52.8	208	113
(3)	16.1	19.5	19.0	19.5	18.8	−14.8	54.8	184	108
(4)	20.5	14.7	16.0	18.3	17.0	−3.7	52.6	205	90
(5)	17.5	19.5	21.9	16.2	18.9	−10.7	50.8	241	152
(6)	18.9	25.1	29.5	28.5	25.9	−20.8	55.7	96	41
(7)	12.9	18.5	19.6	20.6	18.3	−14.0	53.2	239	116
(8)	23.2	20.2	11.9	19.3	19.3	5.0	53.8	256	141
(9)	12.0	11.4	13.5	22.3	15.2	−6.6	52.9	339	191

* The ranges of Ra are: (1) $10^{-8} < Ra \leqslant 10^{-3}$; (2) $10^{-3} < Ra \leqslant 1$; (3) $1 < Ra \leqslant 10^{3}$; (4) $10^{3} < Ra \leqslant 10^{8}$.

† Number of data points predicted to within 10% and 5%.

DEVELOPMENT OF THE NEW CORRELATION

In the conventional analysis of convection problems, the viscous dissipation term in the energy equation is neglected, on the basis of an order-of-magnitude analysis; however, as has been mentioned, Gebhart [9] and Bertela [10] indicate that this neglection is not always justified. The inclusion of the viscous dissipation term leads to the introduction of a dimensionless parameter, which, in the case of forced convection, is the Eckert number, $Ec = V^2/c\Delta t$ (in addition to Nu, Pr and Re); and, in the case of natural convection, is a number defined by $Ge = g\beta l/c$ [9, 24, 25] (in addition to Nu, Pr and Ra). The dimensionless parameter Ge, which will be called the Gebhart number, is the natural convection analog of the Eckert number, if the characteristic velocity V of the natural convection process is taken to be

$$V = (g\beta L\Delta t)^{1/2}. \tag{10}$$

Gebhart calls this value of V "an estimate of convection velocity". Thus, if viscous dissipation is not neglected in natural convection problems, it follows from an examination of the governing set of differential equations that

$$Nu = F(Pr, Ra, Ge). \tag{11}$$

This result was used to provide a starting point for structuring the new correlation that includes the influences of viscous dissipation for natural convection from horizontal cylinders.

The argument that follows is based upon the assumption that the function in equation (11) can be expressed as the sum of two components; thus

$$Nu = f(Pr, Ra) + g(Pr, Ra, Ge). \tag{12}$$

The function $f(Pr, Ra)$ is presumed to represent Nu adequately when viscous dissipation is negligible, and $g(Pr, Ra, Ge)$ is a function that represents a contribution to Nu when viscous dissipation is not negligible.

A study of previously published correlations, followed by a long series of trial-and-error calculations led to the adoption of the following functional forms for $f(Pr, Ra)$ and $g(Pr, Ra, Ge)$ in equation (12):

$$f(Pr, Ra) = aPr^bRa^{1/4} + cPr^dRa^e, \tag{13}$$

$$g(Pr, Ra, Ge) = pPr^qRa^rGe^s, \tag{14}$$

where $a, b, c, d, e, p, q, r, s$ stand for numerical constants to be determined. Computerized nonlinear regression analysis was employed to determine the optimum values of the constants $a, b, \ldots, r, s$ relative to DS1 and DS2 – the values of the constants finally adopted are indicated in equation (9) in Table 1. A number of different functional forms *not involving Ge* were also subjected to nonlinear regression analysis, and for every case considered the exclusion of the viscous dissipation term resulted in an inferior correlation.

DISCUSSION AND CONCLUSION

The data in DS1 is believed to best represent the heat transfer phenomenon under study, hence the ability of an equation for Nu to correlate this data accurately provides the best test of the usefulness of a viscous dissipation parameter in correlating the data. However, the structure of the data set itself has some deficiencies which may limit its usefulness in constructing a general correlation, or adequately determining the role of a viscous dissipation parameter in a general correlation. The most significant of these limitations are: (1) The low Ra data ($Ra < 1$) in DS1 is dominated by air data, while the higher Ra data is dominated by oil data; thus the influences of Pr, Ra and Ge on the variations in Nu are somewhat confounded. (2) Only a limited selection of fluids (five) is represented in DS1 and their Prandtl numbers are not distributed evenly over the entire range of Pr delineated by the data set; this unevenness contributes to the confounding of the influences of Pr, Ra and Ge. These two factors make it difficult to separate the influence of different parameters on the heat transfer phenomenon, and thus correlations constructed on the basis of DS1 do not fully show the potential increase in precision of correlations that include a parameter representing viscous dissipation over correlations that depend on Pr and Ra alone. For example, the (anticipated) larger contribution of viscous dissipation for smaller diameter cylinders and lower values of Ra is confounded in DS1 with a low value of Pr for the data in the low Ra range (i.e. the air data). Hence, it may be that correlations which depend on Pr and Ra alone approximate a "viscous dissipation effect" in the low Ra range with a "Pr effect" in correlating this data set. Equation (7) in Table 1, wherein Nu is expressed as a function of Ra *exclusively*, is an extreme example of the confounding of the influence of one parameter (Ra) with the influences of others (Pr and Ge).

The range of Ge for TDS is $5 \times 10^{-11} \leq Ge \leq 6 \times 10^{-6}$. The magnitude of $g(Pr, Ra, Ge)$ increases with decreases in Pr and Ra. The contribution of $g(Pr, Ra, Ge)$, *expressed as a percentage of Nu*, is approximately 4% for the oil data and 9% for the air data at $Ra = 1$. For the lowest Ra oil data point in DS1 ($Ra = 5.8 \times 10^{-4}$) the viscous dissipation term is 12.5% of Nu, and for the lowest Ra oil data point in DS2 ($Ra = 1.0 \times 10^{-6}$) it is 15.9% of Nu. For the lowest Ra air data point in DS1 ($Ra = 5.7 \times 10^{-5}$) the viscous dissipation term is 34.7% of Nu, and for the lowest Ra air data point in DS2 ($Ra = 4.3 \times 10^{-8}$) it is 51.8% of Nu.

At the present time there is no way to determine conclusively how large, relative to Nu, the viscous dissipation term in equation (12) should be; and it is possible that the contributions of this term calculated above per equation (9) at the lowest reported values of Ra for experiments in air are excessive. If this were so, the error could arise from two primary sources: first, from the aforementioned imbalance in the distribution of the experimental data points in DS1 and DS2 over the ranges of Ra and Pr; and second, from a less-than-optimal choice for the functional forms for $f(Pr, Ra)$ and $g(Pr, Ra, Ge)$. It should be noted that the preceding comment is speculative only with respect to the *relative magnitude* of viscous dissipation, not with respect to its *existence*; for there is no doubt that viscous dissipation occurs in natural convection—that it must be taken into account is suggested by the failure of all previously published correlations (functions of Pr and Ra only) to accurately represent the experimental data in DS2 [within E(rms) = 10%].

Although equation (9) represents the data in DS1, DS2 and TDS more accurately than does any of the previously published equations listed in Table 1, this equation is regarded here merely as a first attempt to take viscous dissipation into account in natural convection. It is expected that this equation will be improved upon in the future as additional data become available, and as improved functional forms for $f(Pr, Ra)$ and $g(Pr, Ra, Ge)$ are developed. Thus, equation (9) is based on functional forms [see equations (12)–(14)] whose terms are products of powers of Pr, Ra and Ge; however, an examination of Table 1 [see particularly equations (1), (2), (3), (6) and (8)] suggests that such simple algebraic forms may not be optimal. Attempts were made in the course of this study to employ simple functional forms involving the logarithms of the independent dimensionless parameters, but these efforts led to results that were inferior to equation (9).

The findings reported here suggest that the inclusion of a viscous dissipation parameter in natural convection situations other than the specific case studied here may lead to more accurate correlations; for example, in the case of natural convection in porous media.

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UNE FORMULE DE TRANSFERT THERMIQUE POUR DES CYLINDRES HORIZONTAUX EN TENANT COMPTE DE LA DISSIPATION VISQUEUSE

Résumé—Huit formulations déjà publiées, plus une nouvelle, pour le transfert thermique par convection naturelle autour de cylindres horizontaux isothermes sont éprouvées par comparaison avec des données expérimentales tirées de la littérature pour $10^{-8} < Ra < 10^{8}$ et $0,7 < Pr < 4.10^{4}$. La nouvelle équation qui représente le nombre de Nusselt en fonction des nombres de Prandtl et de Rayleigh, et d'un paramètre sans dimension lié à la dissipation visqueuse, unifie les données expérimentales avec plus de précision que n'importe laquelle des huits équations déjà publiées. On conclue que la dissipation visqueuse ne peut pas être négligée dans tous les cas de la convection naturelle autour des cylindres horizontaux et que l'inclusion du terme de dissipation dans certains problèmes tels que la convection naturelle dans les milieux poreux peut conduire à des formules plus précises.

EINE BEZIEHUNG FÜR DEN WÄRMETRANSPORT AN HORIZONTALEN ZYLINDERN UNTER BERÜCKSICHTIGUNG DER VISKOSEN DISSIPATION

Zusammenfassung—Acht früher veröffentlichte Gleichungen und eine neuere Gleichung für den Wärmeübergang bei freier Konvektion an horizontalen isothermen Zylindern werden an einer recht umfangreichen Menge experimenteller Daten aus der Literatur für den Bereich $10^{-8} < Ra < 10^{8}$ und $0,7 < Pr < 4 \times 10^{4}$ überprüft. Die neue Gleichung, in der die Nusselt-Zahl eine Funktion der Prandtl- und der Rayleigh-Zahl und eines zusätzlichen dimensionslosen Parameters ist, der die viskose Dissipation berücksichtigt, stimmt mit den experimentell ermittelten Daten besser überein als alle acht früher veröffentlichten Gleichungen. Daraus folgt, daß die viskose Dissipation in allen Fällen der freien Konvektion an horizontalen Zylindern nicht vernachlässigbar ist und daß weiterhin die Einführung eines Terms dür die viskose Dissipation bei einigen ähnlichen Problemen wie der freien Konvektion in porösen Medien zu genaueren Gleichungen führen könnte.

ОБОБЩЕННАЯ ЗАВИСИМОСТЬ ДЛЯ ОПИСАНИЯ ТЕПЛОПЕРЕНОСА ОТ ГОРИЗОНТАЛЬНЫХ ЦИЛИНДРОВ С УЧЕТОМ ВЯЗКОЙ ДИССИПАЦИИ

Аннотация—Одна новая и восемь ранее предложенных зависимостей, описывающих теплоперенос естественной конвекцией от горизонтальных изотермических цилиндров, проверены на достаточно большом объеме экспериментального материала, отобранного из опубликованных работ для $10^{-8} < Ra < 10^{8}$ и $0,7 < Pr < 4 \cdot 10^{4}$. Показано, что новая зависимость, в которой число Нуссельта представлено как функция чисел Прандтля и Рэлея, а также дополнительный безразмерный параметр, учитывающий вязкую диссипацию, более точно описывают экспериментальные данные, чем любое из восьми ранее представленных уравнений. Сделан вывод о том, что при естественной конвекции от горизонтальных цилиндров необходимо всегда учитывать вязкую диссипацию, и что ее учет в некоторых аналогичных задачах, как например, при естественной конвекции в пористых средах, дает более точные результаты.